Analytical Model for Performance Analysis of IEEE 802.11 DCF Mechanism in Multi-Radio Wireless Networks

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***Abstract*—Wireless mesh networks suffer of scalability problems when the number of nodes grows. To solve this issue, wireless mesh networks with multi-interface nodes were introduced. In such networks it is possible to use multiple channels to implement spatial reuse of frequencies. These solutions offer a huge throughput performance improvement but they increase the complexity due to the need of implementing a selection interface policy. One of the most simple interface selection policy is random choice. In this paper we provide an analytical analysis of the Uniform Random Interface Selection strategy applied in a 802.11 DCF multi-radio network. Then we also present a set of performance results for the throughput and discard probability in function of the number of nodes and the number of interfaces.**

***Index Terms*—IEEE 802.11 DCF, multi-radio network, uniform random interface selection algorithm, Basic Access, CTS-to-Self Mechanism**

I. Introduction

Recently IEEE 802.11 mesh networks have been widely deployed during recent year and have received considerable research attention. The absence of a central coordinator produces some interesting consequences, such as: high reliability, easy network maintenance, fast reconfiguration and coverage extension. On the other hand the performance of a mesh network suffer of scalability limits. A transmitting node must share the radio resources with all the other devices in its proximity. Moreover central nodes have a higher packets loss compared to the other. Packets are discarded because the output queues is full or the maximum number of retry is reached. It has been shown that the performance of a single link of a mesh network and in particular of a central node decreases drastically with a number of nodes that form the network [1].

In order to improve network performance, several proposals have been developed based on multi-radio approach in which devices forming the mesh are equipped with more than one radio-interface. These solutions offer a huge throughput improvement but, as a consequence, they increase the complexity due to the need of a policy to decide on which interface the packet must be transmitted.

One of the most simple interface selection policy is the random choice. This approach is not complex but suffers of power consumption. In fact all the interfaces must be always active in RX mode for avoiding to lose packets sent by the neighbours.

In this paper we provide an extended model for analytical analysis of the Uniform Random Interface Selection strategy applied in a 802.11 DCF multi-radio network. We assume an ideal channel condition and a finite number of terminals in the same coverage area (collision domain). Moreover the key approximation that enable our model is the assumption of constant and independent collision probability of a packet transmitted by each station.

The paper is outlined as follows. In Section II we introduces the generalized Markov model refering to [2] [4]. In Section III we describes the DCF model for the Uniform Random Interface Selection strategy. Finally, in Section IV we analytically determines the relevant performance measures in terms of Normalized Throughput and Discard Probability for a non saturated 802.11g homogeneous network and heterogeneous 802.11g and 802.11b network.

II. DCF Generalized Model

There are some good works dedicated their research in establishing theoretic analysis model of IEEE 802.11 DCF mechanism.

Bianchi [2] presents a Markov model where each station is modeled by a pair of integer stochastic processes $\{i(t), k(t)\}$. $i(t)$ represents the current value of the station's back-off stage: it starts at 0 at the first attempt to transmit a packet and is increased by 1 every time a transmission attempt results in a collision, up to a maximum value m. It is reset after a succesful transmission. Parameter $k(t)$ represents the back-off counter that is uniformly chosen in the range $[0, W_i - 1]$ for a given stage $i(t)$. W_i is the current value of the station contention window (CW) that depends on the number of previous consecutive unsuccessful retransmission attempts, i.e.

$$W_i = W \cdot 2^i, \ 0 \leq i \leq m \qquad (1)$$

In (1), W is the minimal value of the contention window (CW_{min}) and m is the number of retransmissions during the exponential increase of the backoff window. Transmission is attempted when $k = 0$.

The Markov model in [3] proposes an additional row of so called *e*-state on top of the Bianchi's bidimensional chain [2] to manage 802.11 CSMA/CA mechanism under a non-saturated traffic assumption. This new state denote a kind of a virtual back-off counter initiated prior to packet arrival: if no packet arrives in the transmission queue (occurring with probability $1-q$), the virtual counter decrement; if a packet arrives (occurring with probability q), the virtual counter becomes a real counter and resumes its normal operation. Actually, when being in the *e*-state, station executes the *collision avoidance* (CA) procedure prior to the arrival of the next packet in the transmission queue. Such defined *e*-state assure the statistical stability of the model while not changing the station's DCF operation as specified by the IEEE 802.11 standard.

The essential assumption for independence and invariance of the conditional collision probability p is also preserved here. This probability, as shown in (2), can be expressed in terms of the transmission probability τ of any station in a randomly chosen time slot and the number of stations n in the network. Consequently, the medium is idle with probability $p_{idle} = 1-p = (1-\tau)^{n-1}$. Hence,

$$p = 1-(1-\tau)^{n-1} \tag{2}$$

Under these assumptions, the transition probabilities from *e*-state to the rest of the possible states are shown in Fig. 1

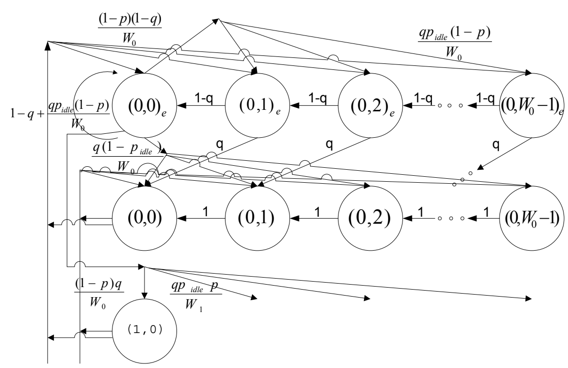

Fig. 1. Additional transition probabilities in the Markov Chain Model in [3]

Note that the state $(0,0)_e$ is the most complex. If a packet arrives, the transition to a new state depends on the current state of the medium: if the medium is idle, transmission may be attempted, where collision (see 5) or successful transmission can occur (see 3 and 4); if the medium is busy, the classic CA is initiated by transition into an arbitrary state $(0,k)$ (6).

$$P[(0,0)_e|(0,0)_e] = 1-q+\frac{qp_{idle}(1-p)}{W_0} \tag{3}$$

$$P[(0,k)_e|(0,0)]_e = \frac{qp_{idle}(1-p)}{W_0} \tag{4}$$

$$P[(1,k)_e|(0,0)]_e = \frac{qp_{idle}p}{W_1} \tag{5}$$

$$P[(0,k)|(0,0)]_e = \frac{q(1-p_{idle})}{W_0} \tag{6}$$

The Markov chain in [4] extends the previous model because it takes into account, in addition to the existing m retries, a finite number of (re)transmission attempts f with a fixed backoff window (2^mW) after that a packet is discarded. The transition probabilities, rispectively to the *e*-states $(0,k)_e$ and to the *zero-row* states $(0,k)$ from the state $(m+f,0)$ are given (see Fig. 2) by:

$$P[(0,k)_e|(m+f,0)] = \frac{(1-q)}{W_0} \tag{7}$$

$$P[(0,k)|(m+f,0)] = \frac{q}{W_0} \tag{8}$$

After $m+f$ unsuccessful (re)transmission attempts, if the transmission queue is empty, the station moves into one of the *e*-states with transition probability given by (7), instead if it is non empty, the station moves into one of the *zero-row* states with probability shows in (8). Solving for the stationary

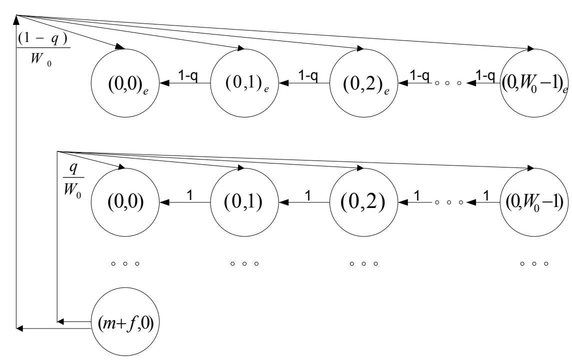

Fig. 2. Additional transition probabilities in the Markov Chain Model in [4]

distribution of the bidimensional chain, we can calculate the transmission probability τ:

$$\tau = \frac{1}{\eta}\frac{q^2}{1-q}\left[\frac{W_0}{(1-p)(1-(1-q)^{W_0})} - (1-p)\right] \tag{9}$$

where,

$$\eta = \frac{1}{1-p^{m+f+1}}\left\{A+B+C-D\right\}$$

$$A = (1-q)+\frac{q^2W_0(W_0+1)}{2(1-(1-q)^{W_0})}$$

$$B = \frac{q(W_0+1)}{2(1-q)}\left(\frac{q^2W_0}{1-(1-q)^{W_0}}+p(1-q)-q(1-p)^2\right)$$

$$C = \frac{pq^2}{2(1-q)(1-p)}\left(\frac{W_0}{1-(1-q)^{W_0}}-(1-p)^2\right)\left(2W_0\frac{1-p-p(2p)^{m-1}}{1-2p}+1\right)$$

$$D = \frac{1+W_02^m}{2}\frac{q^2p^{m+f+1}}{1-q}\frac{W_0-(1-p)^2(1-(1-q)^{W_0})}{(1-p)(1-(1-q)^{W_0})}$$

Assuming that the inter-packet arrival times are exponentially distributed, q can be expressed as follow:

$$q = 1-e^{-\lambda E} \tag{10}$$

where λ is the arrival rate and E is the average length of a slot time. E can be computed as shown in [2].

Therefore, in order to characterize analytically the overall model in terms of transmission probability, collision probability and packet arrival probability, it is necessary to solve a non-linear equation system formed by (2) (9) (10) with three unknowns (τ, p, q).

III. DCF Model for Uniform Random Interface Selection

One of the fundamental assumption on which the DCF model described in previous subsection is based, is the time invariance of the network parameters (i.e., arrival rate, collision domain). This condition does not permit to provide for most cases an extended analytical model for CSMA-CA channel access mechanism in presence of stations with more than one radio interface; in fact the interface selection strategy, exstablished a priori, could modify over time the behaviour of the network. However, if the basic interface selection policy is uniform random choice, it is possible to customize the single interface DCF model. In this case network parameters doesn't dipend on the selected interface.

Consider a number n of stations in the same coverage area, each having N radio interfaces.

Let $P_i = (p_i(1), p_i(2), ..., p_i(N))$, where $\sum_{j=1}^{N} p_i(j) = 1$, the switching vector for each station $i = 1, ..., n$; in the particular case of an uniform random interface selection strategy, it assumes the value of $P_i = (1/N, 1/N, ..., 1/N)$.

Since only one interface per time can be selected to transmit a packet, $\frac{n}{N}$ stations contend on the channel; therefore the collision probability results to be:

$$p = 1 - (1-\tau)^{\frac{n}{N}-1} \tag{11}$$

Instead, since all the avaiable interfaces are at the same time in listening mode, the packet arrival probability is given by the probability that there is at least one packet in at least one interface:

$$\begin{aligned} q &= 1 - (\overbrace{exp(-\lambda E) exp(-\lambda E) \cdots exp(-\lambda E)}^{N \text{ times}}) \\ &= 1 - exp(-\lambda N E) \end{aligned} \tag{12}$$

where λ is the arrival rate and E is the average length of a slot time. E is also a function of τ and is readily obtained considering that:

- the slot time (σ) is empty with probability $1 - P_{tr}$,
- the slot time (σ) contains a successful transmission with probability $P_{tr} P_s$,
- the slot time (σ) contains a collision with probability $P_{tr}(1 - P_s)$.

Hence E becomes:

$$E = (1 - P_{tr})\sigma + P_{tr} P_s T_s + P_{tr}(1 - P_s) T_c \tag{13}$$

P_{tr} is the probability that there is at least one transmission in the considered slot time:

$$P_{tr} = 1 - (1-\tau)^{\frac{n}{N}} \tag{14}$$

P_s is the probability that a transmission occuring on the channel is successful and is given by the probability that exactly one station transmits on the channel.

$$P_s = \frac{(n/N)\tau(1-\tau)^{\frac{n}{N}-1}}{P_{tr}} \tag{15}$$

T_s and T_c are respectively the average time the channel is sensed busy because of a successful transmission and the average time the channel is sensed busy by each station during a collision. Their values depend on the IEEE 802.11 standard and the channel access mechanism adopted (i.e., Basic Access, RTS/CTS Access, CTS-to-Self Access).

Therefore, in order to model analytically the system, it is necessary to solve the non-linear equation system formed by (9) (11) and (12) with three unknowns (τ, p, q).

IV. Numeric Performance Evaluation

To evaluate the performance of the uniform random interface selection strategy in a multiradio network, some analytical simulations have been performed by using the DCF model described above. The non linear equation system has been solved by a MATLAB framework called TRESNEI [5] [6].

The values of the network parameters, used to obtained numerical results, are summarized in TABLE I [7] [8] [9]. They are those specified for two typical scenarios according to IEEE 802.11g standard: an homogeneous 802.11g WLAN and an heterogeneous 802.11b and 802.11g WLAN.

In the first scenario the 802.11g complying devices transmit using the ERP-OFDM modulation scheme without the use of any protection mechanism, unless there is the need for the reservation of the channel or the avoidance of the hidden node problem. The Basic Access involves a two-way message exchange that consists in sending DATA and waiting for the ACK. The maximum DATA length frame is a series of 12288 $bits$ plus 6 tail $bits$[1] encoded into 57 symbols. The ACK message instead requires just one symbol of 216 $bits$.

In the second scenario the 802.11g complying devices always transmit using the ERP-OFDM modulation scheme. As the non-802.11g devices can not detect the ERP-OFDM messages, there is a need to introduce a Protection Mechanism. For the purpose of this work a CTS-to-Self Access has been considered. The IEEE 802.11g standard defines the CTS-to-Self Access as an alternative to RTS-CTS in order to reduce the overhead added in WLAN system. Unlike RTS/CTS, CTS-to-Self can not efficiently face the hidden terminal problem. The CTS-to-Self Access is exactly the same of the Basic Access except for the introduction of a 112 $bits$ CTS message trasmitted at one of the 802.11b rates before a DATA packet. Also other parameters, like for example the slot time and the DIFS interval, are fit to 802.11b technology.

The next subsections present the numerical and the performance analysis results for the two scenarios described above in terms of Normalized Throughput and Discard Probability.

[1]The ERP-OFDM adds 6 $bits$ for encoding purpose to the end of the frame

TABLE I
ERP-OFDM SYSTEM PARAMETERS AND MAC PARAMETERS FOR A BASIC ACCESS AND CTS-TO-SELF ACCESS USED TO OBTAIN NUMERICAL RESULTS

	Basic Access	CTS-to-Self Access
Packet Payload	1500 *byte*	1500 *byte*
MAC Header	36 *byte*	36 *byte*
ACK Payload	27 *byte*	27 *byte*
CTS Payload	—	14 *byte*
Bit Rate	54 $Mbps$	54 $Mbps$
Arrival Rate	32 $Mbps$	32 $Mbps$
CW_{min}	15 μs	15 μs
Propagation Delay	1 μs	1 μs
Slot Time	9 μs	20 μs
SIFS	10 μs	10 μs
DIFS	28 μs	50 μs

TABLE II
TIME PACKETS VALUES FOR BASIC ACCESS AND CTS-TO-SELF ACCESS

802.11 Data	20 μs + 57 $*$ 4 $\mu s/symbol$ + 6 μs = 20 μs + 228 μs + 6 μs = 254 μs
ACK	20 μs + 1 $*$ 4 $\mu s/symbol$ + 6 μs = 20 μs + 4 μs + 6 μs = 30 μs
CTS (only for CTS-to-Self Access)	192 μs + 14 $byte$/ 11 $Mbps$ = 192 μs + 11 μs = 203 μs

A. Normalized Throughput

Let S the normalized system throughput, defined as the fraction of time the channel is used to successfully transmit payload bits. It can be expressed as the ratio

$$S = \frac{E[payload\,information\,transmitted\,in\,a\,slot\,time]}{E[length\,of\,a\,slot\,time]} = \frac{P_s P_{tr} E[P]}{(1-P_{tr})\sigma + P_{tr}P_sT_s + P_{tr}(1-P_s)T_c} \quad (16)$$

Note that throughput defined in (16) has been obtained without the need to specify the access mechanism employed. To specifically compute the throughput for a given DCF access mechanism it is now necessary only to specify the corrisponding value of T_s and T_c. (17) and (18) show respectively the estimation of T_s and T_c for a Basic Access and a CTS-to-Self Access.

$$\begin{aligned} T_s &= H + E[P] + SIFS + \delta + ACK + DIFS + \delta \\ T_c &= H + E[P] + DIFS + \delta \end{aligned} \quad (17)$$

$$\begin{aligned} T_s &= CTS + SIFS + H + E[P] + SIFS + \delta \\ &= +ACK + DIFS + \delta \\ T_c &= CTS + DIFS + \delta \end{aligned} \quad (18)$$

δ is the propagation delay, $E[P]$ is the average packet payload length and $H = PHY_{hdr} + MAC_{hdr}$ is the packet header length.

In Fig. 3 and in Fig. 4 the Normalized Throughput as a function of the number of network nodes and the number of radio interfaces is presented in both the cases of Basic Access and CTS-to-Self mechanism.

Both the figures show that the throughput strongly depends on the number of stations in the network. The presence of only one absolute peak in each curve progress indicates that the optimal performance are achievable for a fixed number of stations in the network since the backoff window (W) and the number of retransmissions (m) are hardwired in the 802.11g PHY layer. In particular in Fig. 4 the greater is the network size, the lower is the throughput; it means that the network is already saturated for a small number of contending stations. Instead the Fig. 3 shows that the same behaviour does not occur in small size networks for a Basic Access. In fact the throughput trend increases until the achievement of the best performance. It means that in this interval there is a resource redundancy.

A remarkable performance gain in terms of throughput provided by the introduction of more than one radio interface also could be noticed: in particular, it increases at the increasing of the node density, underlying an optimal scalability.

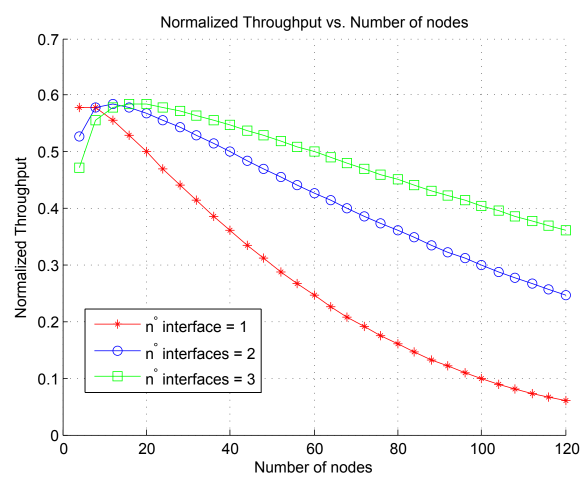

Fig. 3. Normalized Throughput vs. Number of Nodes for the Basic Access

Finally, the CTS-to-Self Access, introduced to prevent collisions caused by the DSSS-OFDM interoperatibility problem, drops dramatically the throughput relative to the Basic Access

B. Discard Probability

After a packet suffers $m + f + 1$ consecutive unsuccessful retransmission attempts, this packet is discarded from the transmission queue and a transmission for another packet is attempted immediatily (if any in the transmission queue) or latter through the back-off phase. Thus, the discard probability can be calculated as:

$$P_d = p^{m+f+1} \quad (19)$$

In Fig. 5 and in Fig. 6 the Discard Probability as a function of the number of network nodes and the number of radio interfaces is presented in both the cases of Basic Access and CTS-to-Self mechanism.

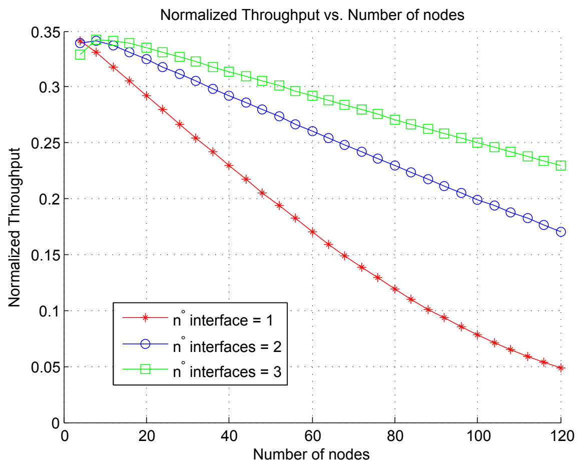

Fig. 4. Normalized Throughput vs. Number of Nodes for the CTS-to-Self Mechanism

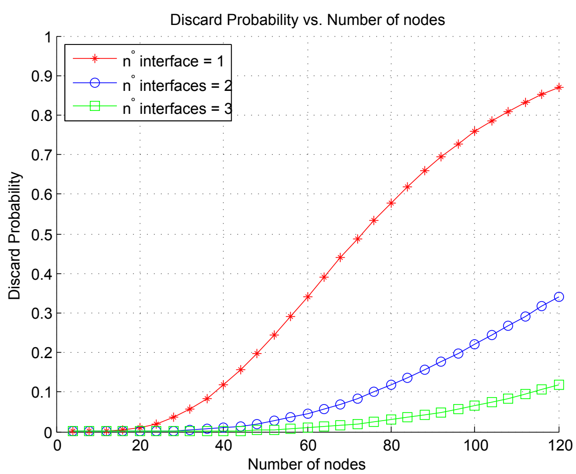

Fig. 5. Discard Probability vs. Number of Nodes for the Basic Access

By increasing the network size, the discard probability increases towards the 100%. Also it could be noticed that the adoption of more than one radio interface minimizes the discard packets and improves consequently the network throughput since the busy channel is minimized.

V. Conclusion

In this paper, on the analytical basis of the other previous research work, we have presented an analytical model to compute the performances, in terms of throughput and discard probability, of the the Uniform Random Interface Selection strategy applied in a 802.11 DCF multi-radio network. We have assumed an ideal channel condition and a finite number of terminals in the same coverage area (collision domain).

The performances have been evaluated for two typical scenarios: an homogeneous 802.11g WLAN and an heterogeneous 802.11b and 802.11g WLAN.

The results show the performance improvement obtained by the use of more than one radio interface in both the scenarios.

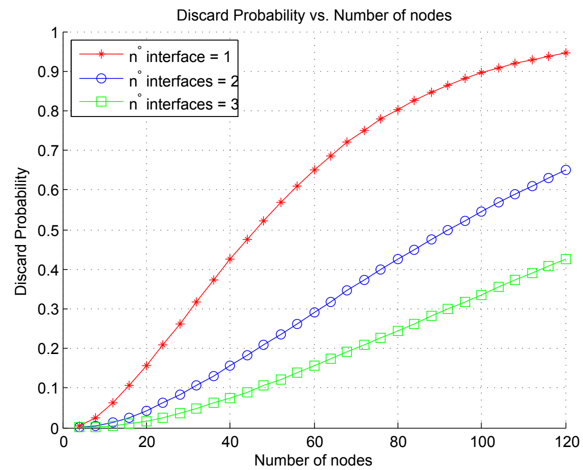

Fig. 6. Discard Probability vs. Number of Nodes for the CTS-to-Self Mechanism

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