# Evacuating Robots from an Unknown Exit in a Disk

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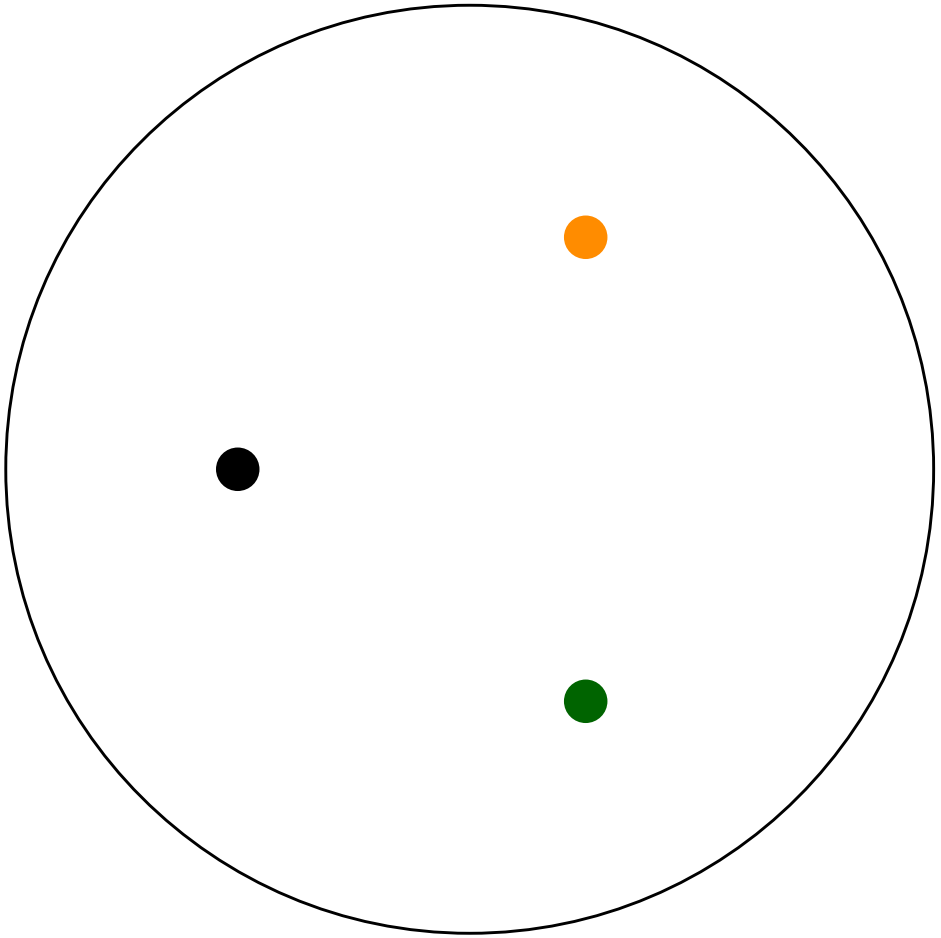
# Evacuation problem



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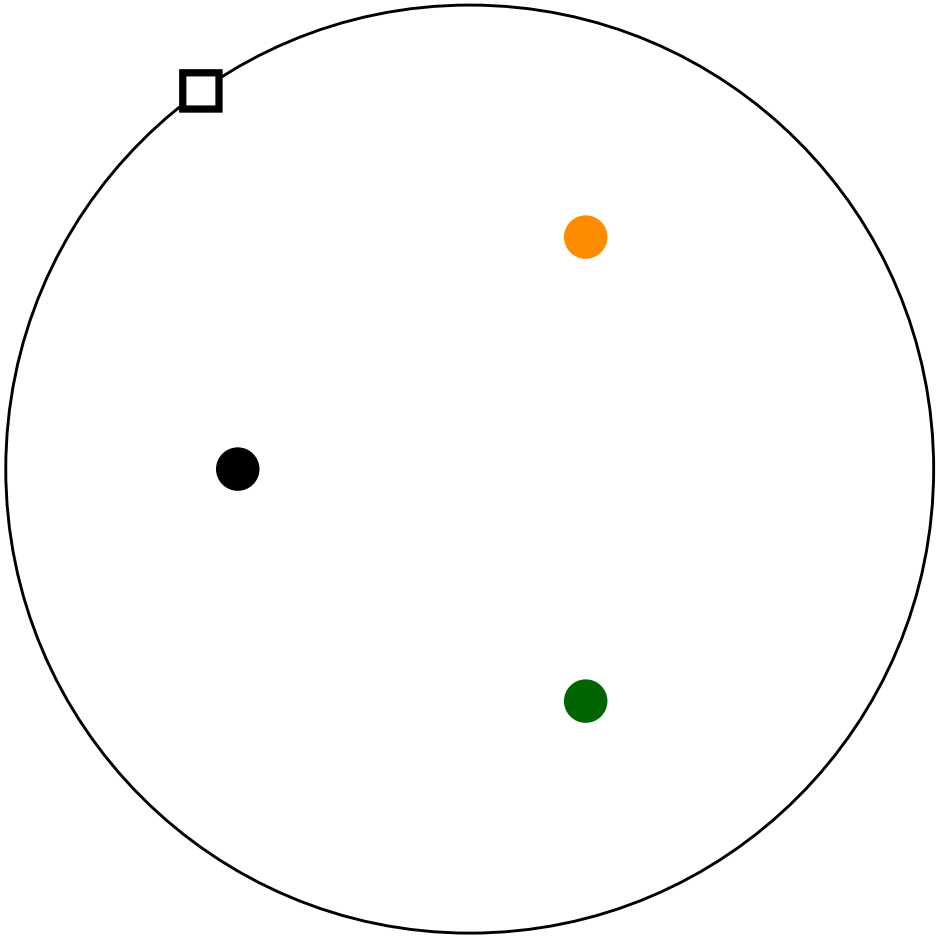

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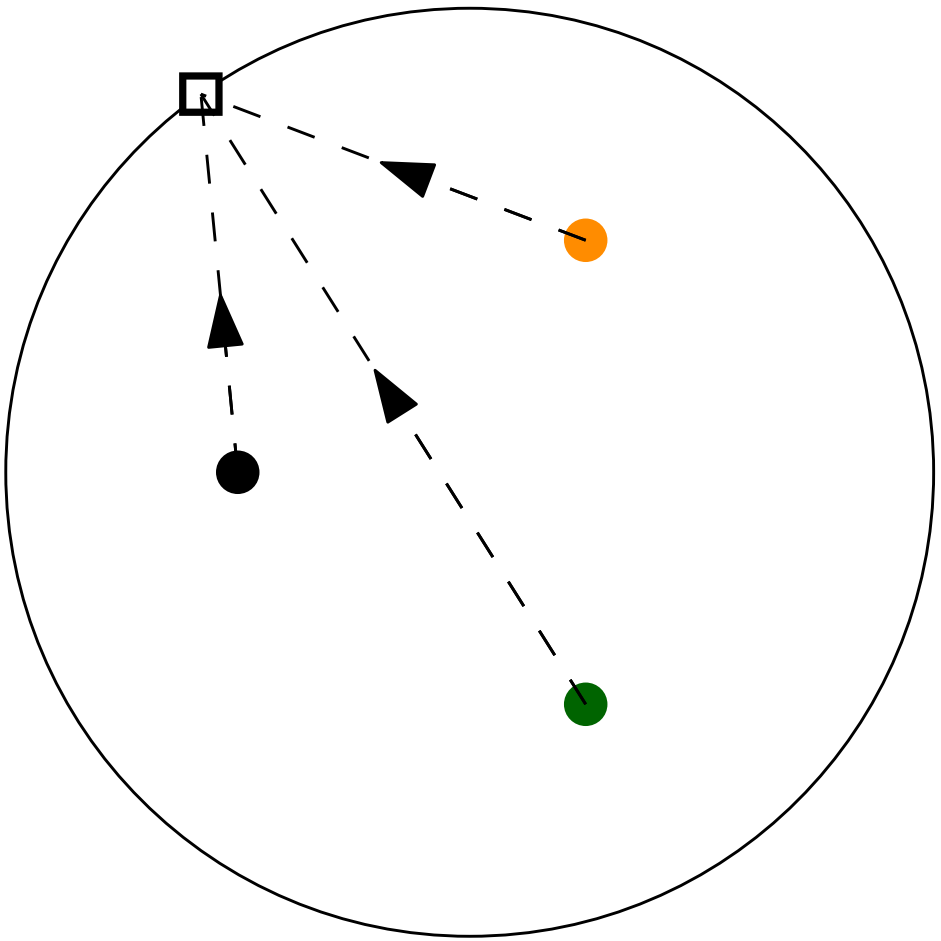

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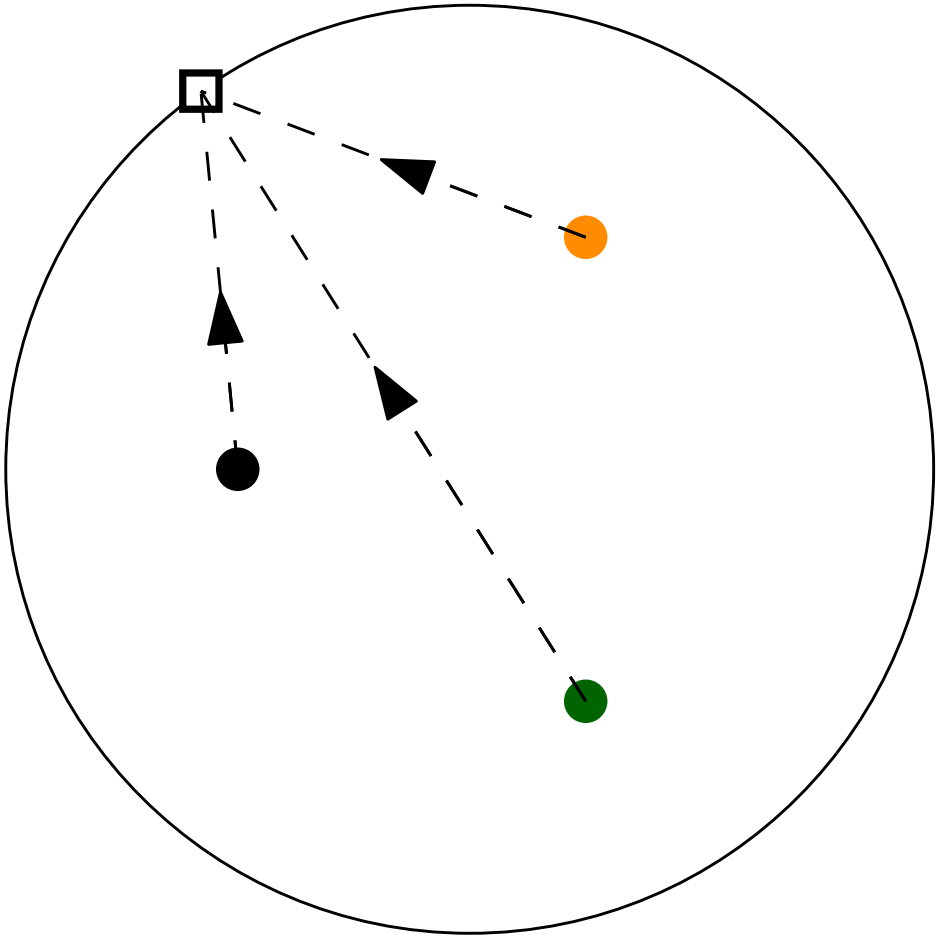

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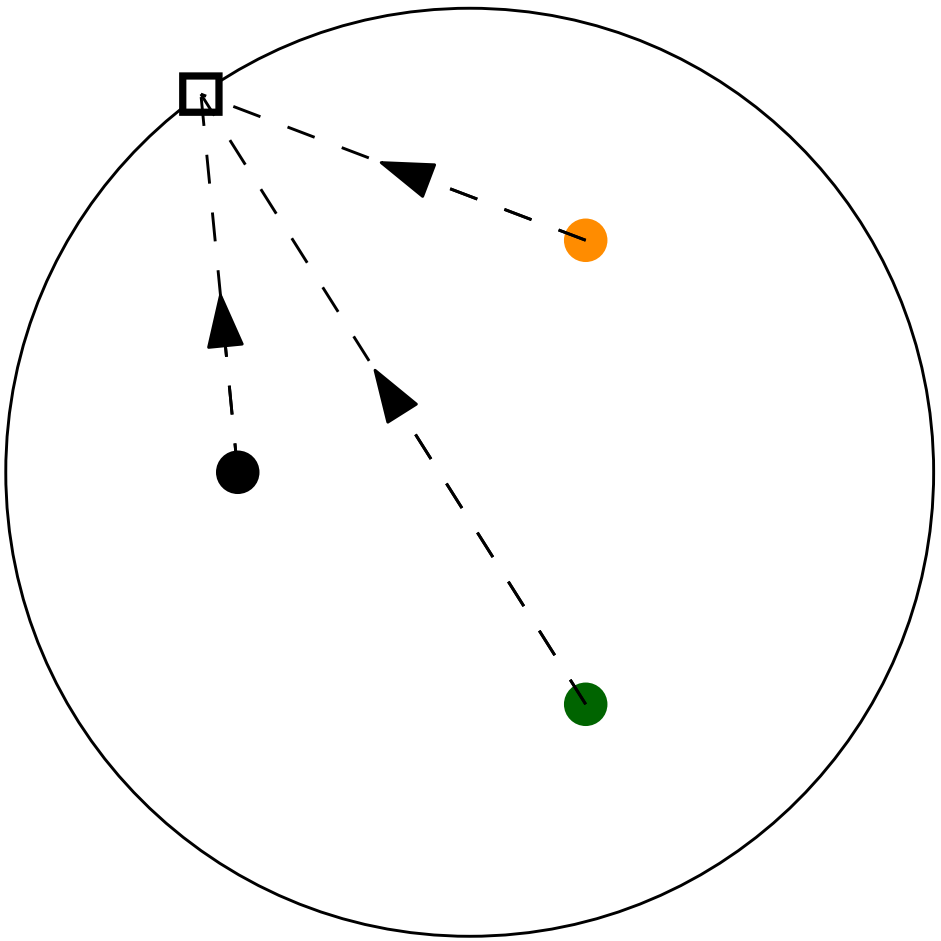

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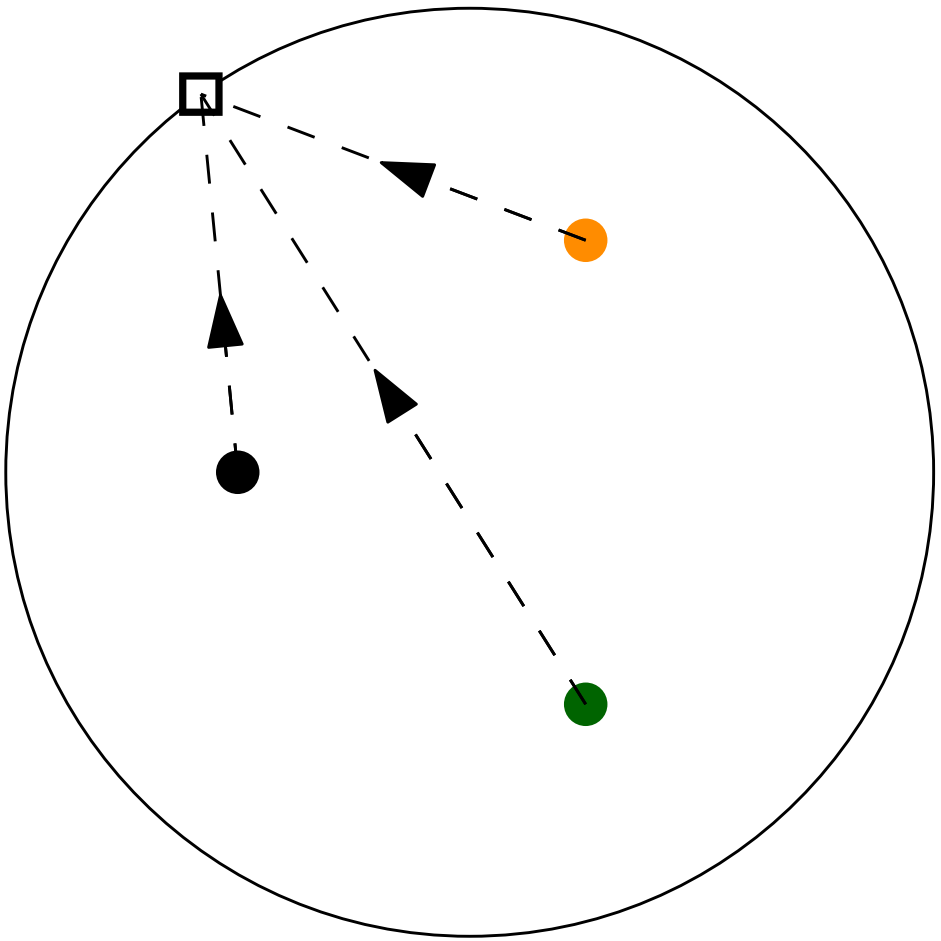

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- Exit is placed by an adversary. Location of the exit is **unknown** to the robots.
- Robots can move inside the disk with unit speed.
- Evacuation time is the time when the **last** robot arrives at the exit.
- **Goal:** minimize the evacuation time.

# Communication

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- Some means of communication is necessary to achieve speedup.

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## Communication models

- **Wireless** – robots can exchange information at any place/time.
- **Non-wireless** (also called **face-to-face**)– robots can communicate only when located at the same point.

# Details of the model

## No visibility

- Cannot see other robots or the exit from a distance.
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- Distance to the exit is always 1.

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## Deterministic

- A robot that has no information about exit follows a fixed trajectory.

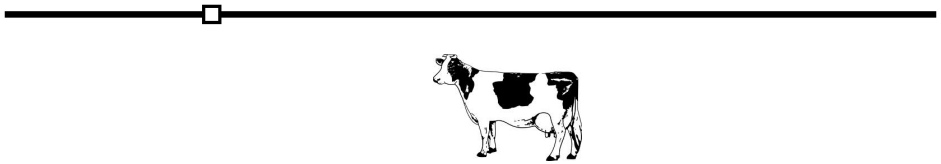
# Related work – evacuation on a line

- Evacuation of one robot is equivalent to the cow-path problem.

**Cow-path problem**

The problem of searching for an exit point on an infinite line.

- $d$ – distance to the exit from the starting position.
- Simple doubling strategy guarantees finding the in time $9d$.
- There is no better deterministic strategy.



# Variant of the cow path problem

- The doubling strategy has the problem that at the beginning it needs to make an infinite number of infinitesimal steps.
  - Not a problem if you have a "unit".
- An interesting (and practical) modification is to introduce a cost $\delta$ for each turn.
- Then there exists strategy (very similar to simple doubling) that has only a finite number of turns and the optimal search time is $9d + 2\delta$

  [Demaine, Fekete, Gal 2006]

- Other variants – for example star search.

# Related work – evacuation on a line

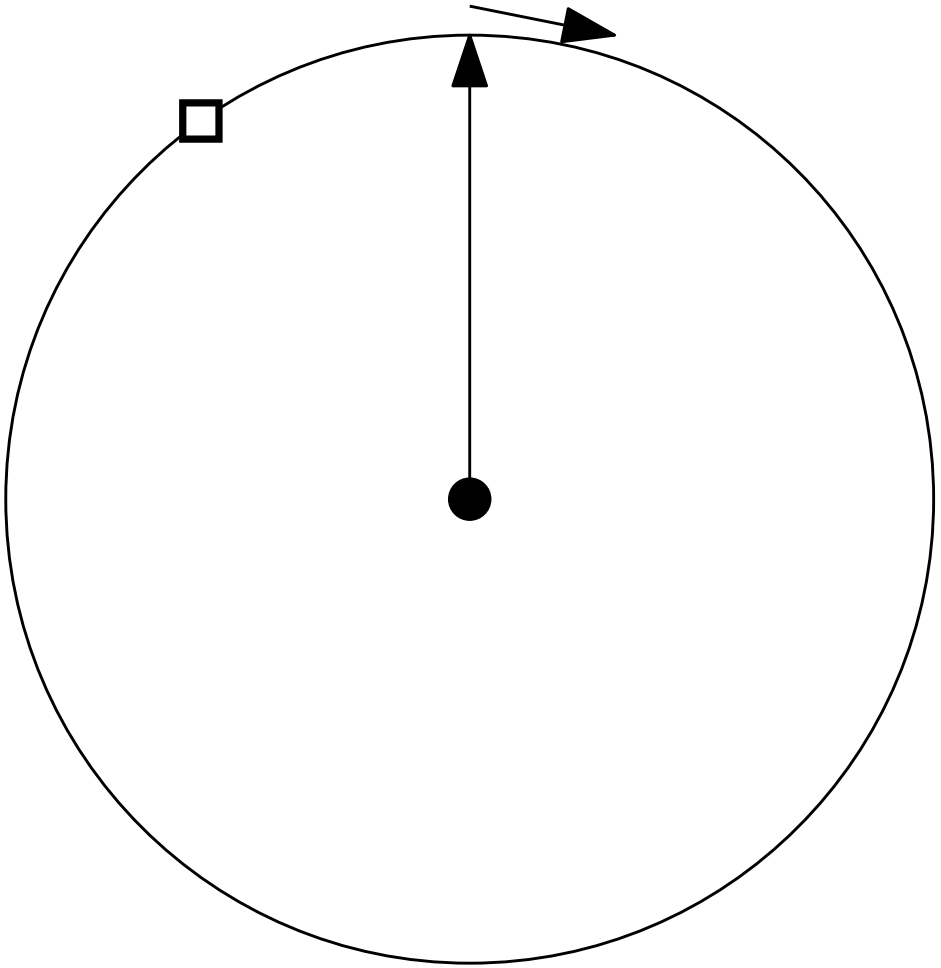
## Evacuation of multiple robots on a line.

- $9d$ is optimal evacuation time of multiple robots with the same speed on a line.
- For multiple robots with different speeds $v_1 \leq v_2 \leq \ldots v_k = 1$ it is still possible to evacuate in time $9d$ as long as the minimum speed satisfies: $v_1 \geq 1/3$. [Chrobak, Gąsieniec, Gorry, SOFSEM 2015]

## *Exercise:* Evacuation of two robots on a line.

- Two robots with same speed start at the same point on a line.
- How long does it take to evacuate in the
  - wireless model?
  - non-wireless model?

# Single Robot: a simple example



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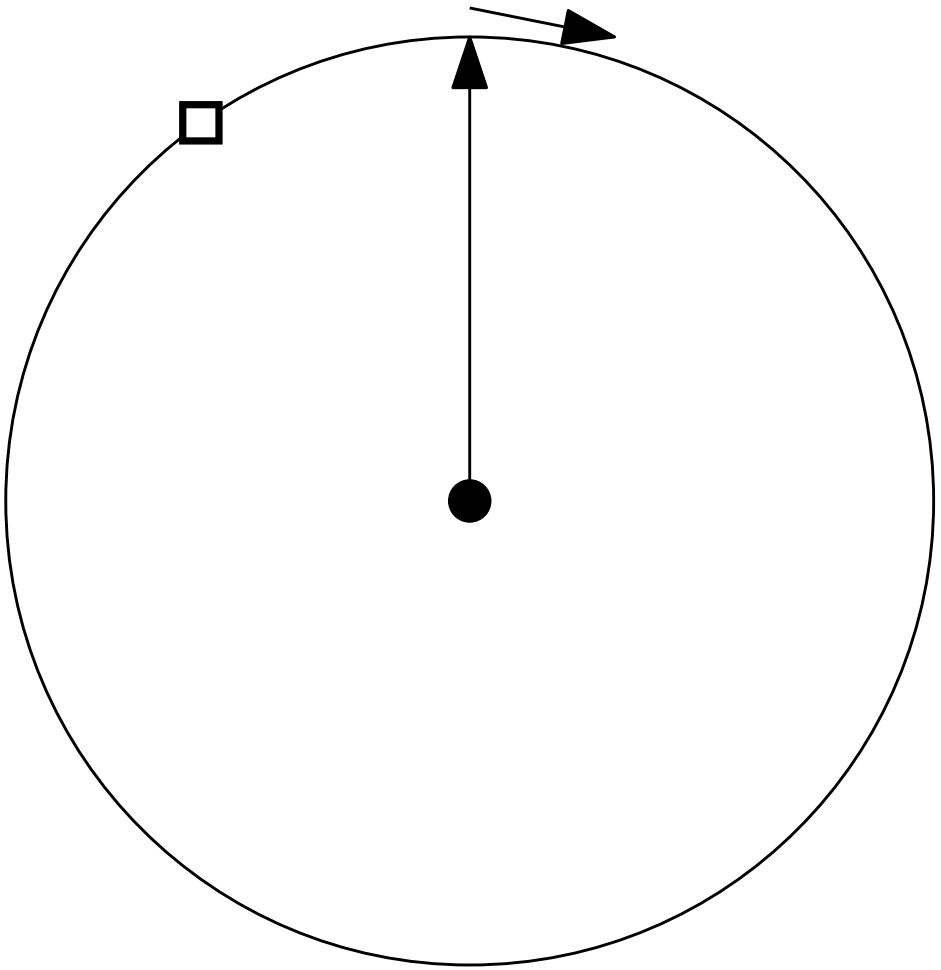

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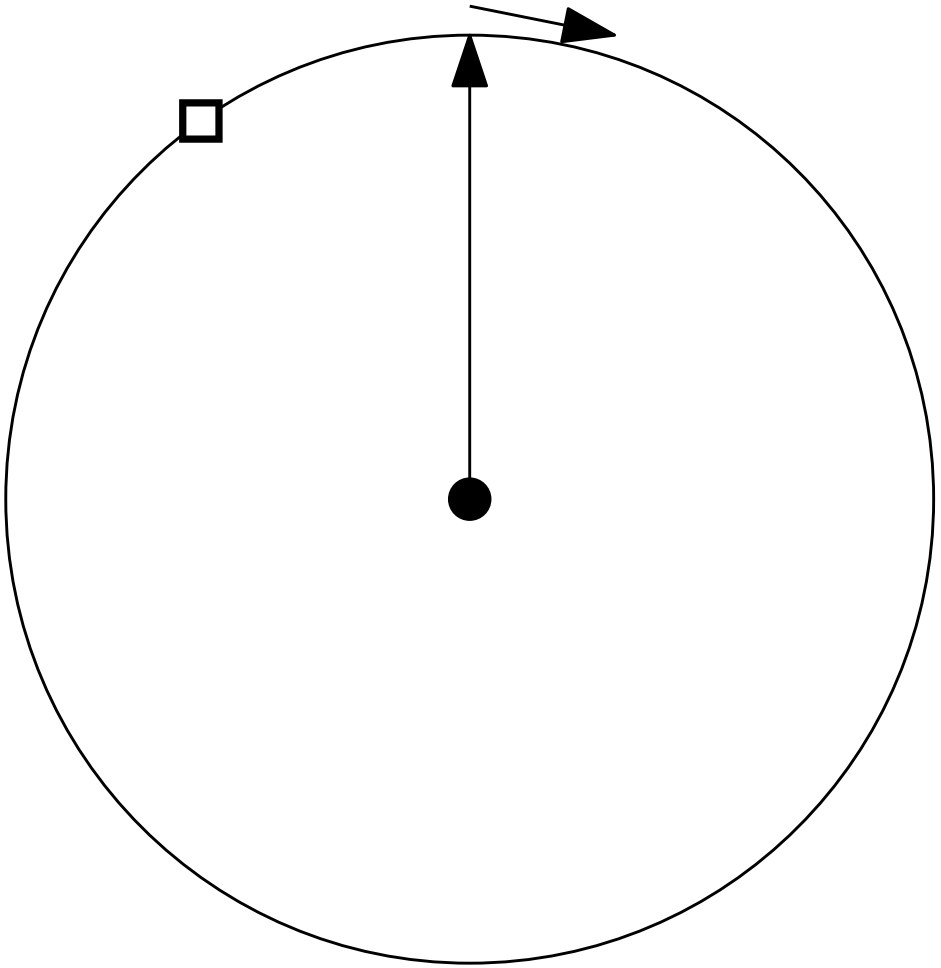

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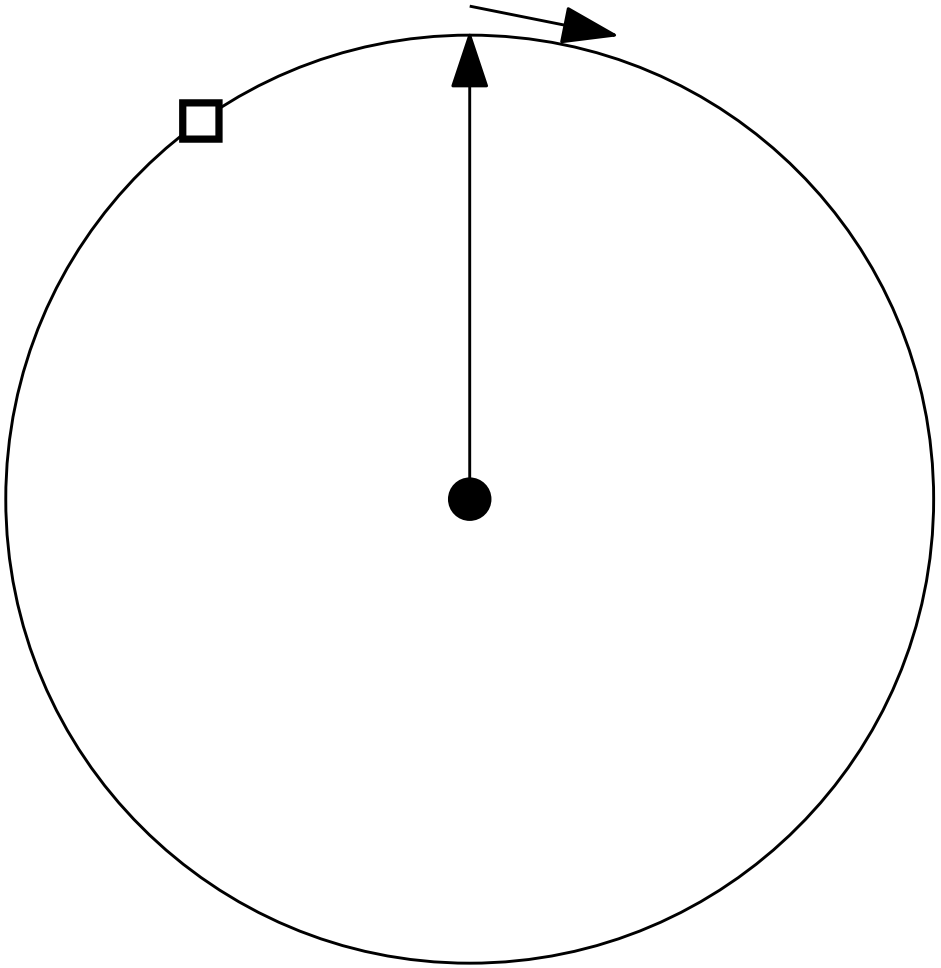

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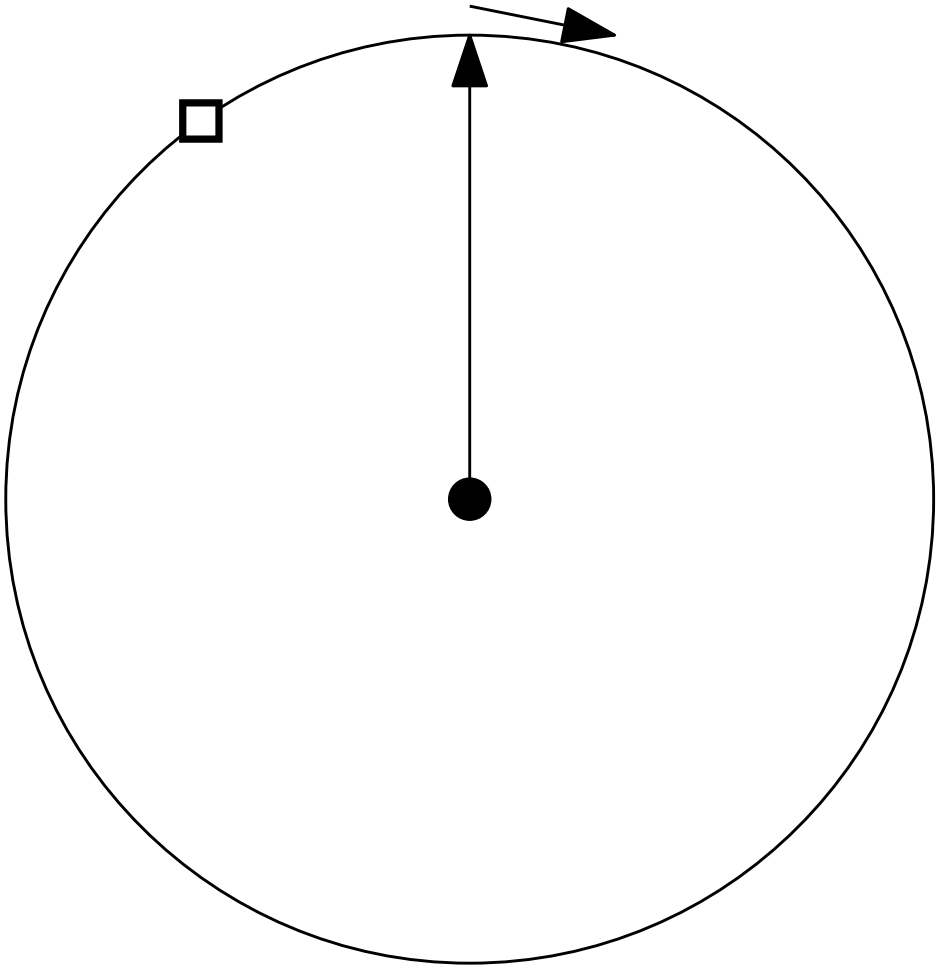

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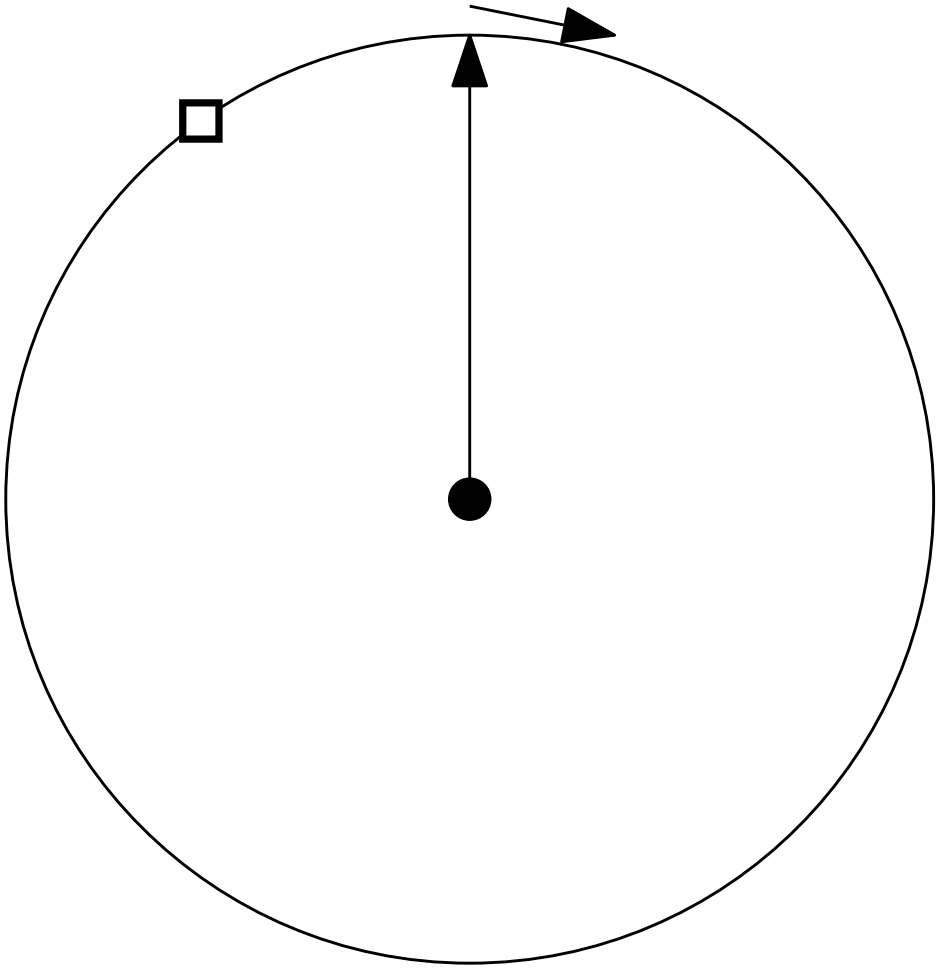

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- Evacuation time is $1 + 2\pi$.

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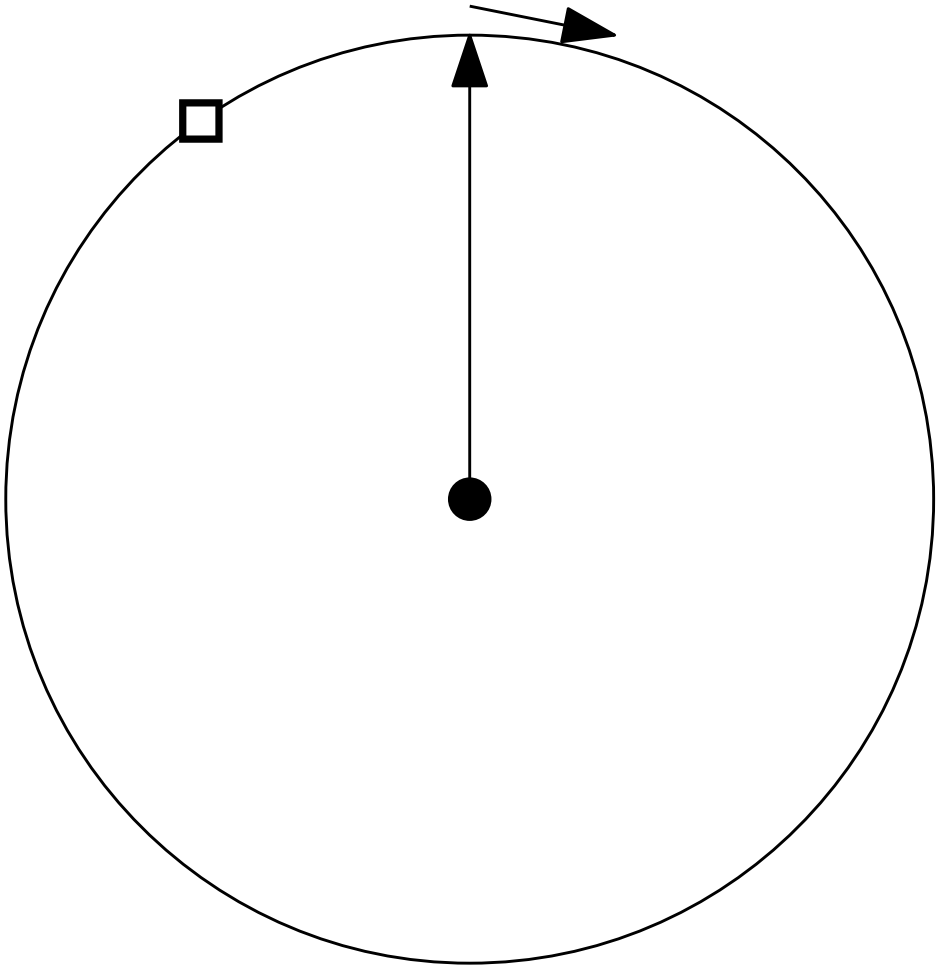

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- Robot must traverse the whole perimeter.
- Evacuation time is $1 + 2\pi$.
- $1 + 2\pi$ is also a lower bound.

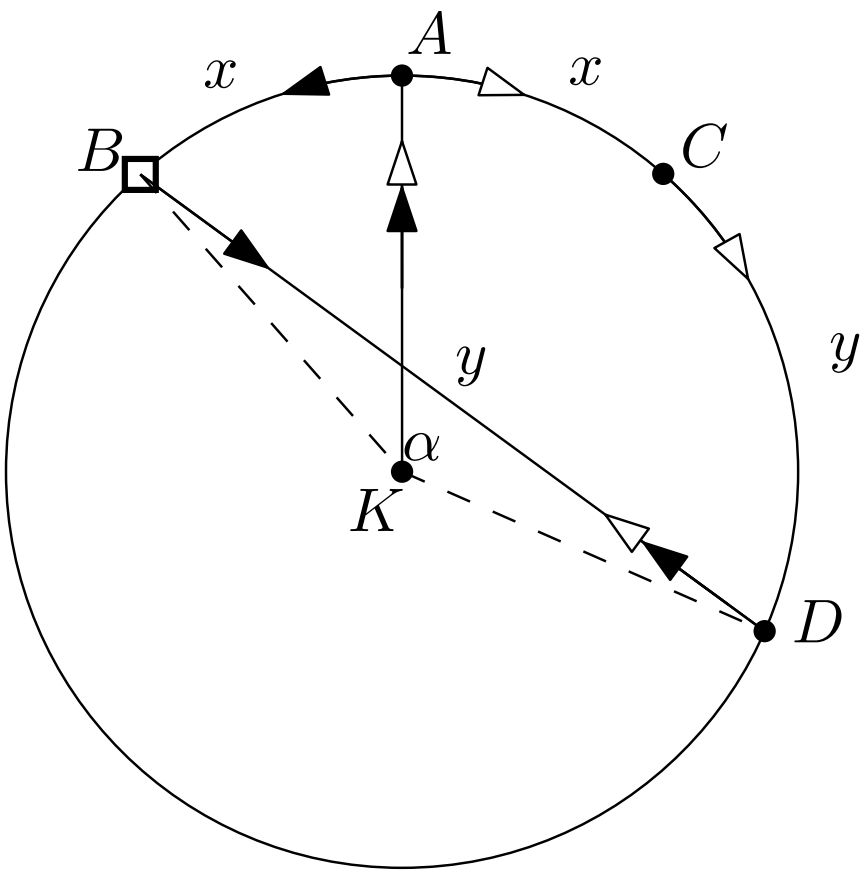
# Results

| *Communication* | | *Upper bound* | *Lower bound* |
|---|---|---|---|
| Non-wireless | $k = 2$ | $\sim 5.74$ | $\sim 5.199$ |
| Wireless | $k = 2$ | $1 + \frac{2\pi}{3} + \sqrt{3} \sim 4.83$ | $1 + \frac{2\pi}{3} + \sqrt{3} \sim 4.83$ |
| Non-wireless | $k = 3$ | $3 + \frac{2\pi}{3} \sim 5.09$ | $\sim 4.519$ |
| Wireless | $k = 3$ | $\sim 4.22$ | $\sim 4.159$ |
| Non-wireless | large $k$ | $3 + \frac{2\pi}{k}$ | $3 + \frac{2\pi}{k} - O(k^{-2})$ |
| Wireless | large $k$ | $3 + \frac{\pi}{k} + O(k^{-4/3})$ | $3 + \frac{\pi}{k}$ |

# $k = 2$ summary

| *Communication* | *Conferebce* | *Upper bound* | *Lower bound* |
|---|---|---|---|
| Non-wireless | DISC 2014 | 5.744 | 5.199 |
| | CIAC 2015 | 5.628 | 5.255 |
| Wireless | DISC 2014 | $1 + 2\pi/3 + \sqrt{3}$ | $1 + 2\pi/3 + \sqrt{3}$ |

# Non-wireless, $k = 2$, algorithm




## Algorithm

- Both robots go to the point $A$.
- Robots go in the opposite directions.
- When a robot discovers the exit, it takes a chord to meet the other robot and bring it to the exit.

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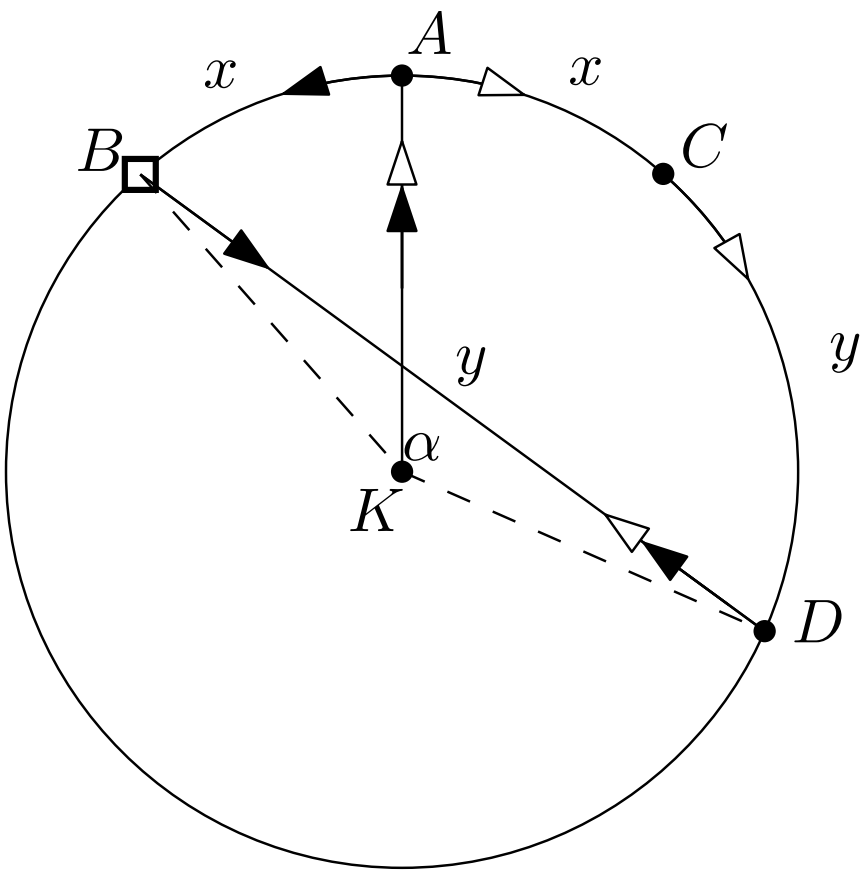


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- Evacuation time $T = 1 + x + 2y$.
- $\alpha$ is the angle from $B$ to $D$, $\alpha = 2x + y$.
- $y$ is length of the cord on arc $\alpha$, thus $y = 2\sin(\alpha/2)$
- $T = f(\alpha) = 1 + \alpha/2 + 3\sin(\alpha/2)$.
- $\max_\alpha f(\alpha) \approx 5.74$.

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- If it does not, it returns to the perimeter and continues.

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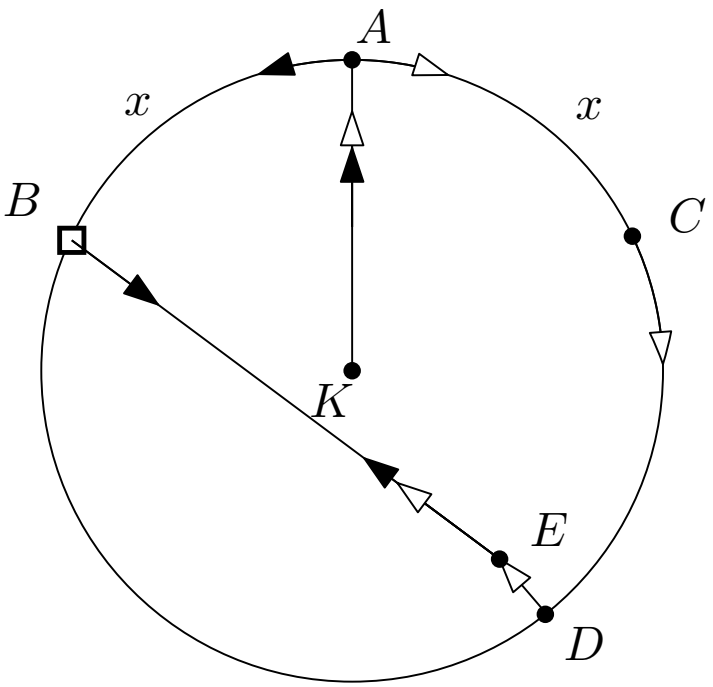


- Algorithm can be improved (to $\sim 5.628$) using a *very strange* strategy.

# Non-wireless, $k = 2$, lower bound

**Theorem**

It takes at least time $3 + \pi/4 + \sqrt{2} \approx 5.199$ for two robots to evacuate in the non-wireless model.

**Proof**

- Within time less than $1 + \pi/4$ less than $\pi/2$ of the perimeter is explored thus there exists a square with all vertices unexplored.
  - To see this think about rotating a square inscribed inside the circle by angle $\pi/2$ – one of the vertices has to be explored all the time during the rotation.
- It is left to show that it takes $2 + \sqrt{2}$ to evacuate from a square with side of length $\sqrt{2}$, regardless of the positions of robots.
  - Easy case analysis.

Optimal strategy is unknown!

# Wireless, $k = 2$, algorithm

## Algorithm

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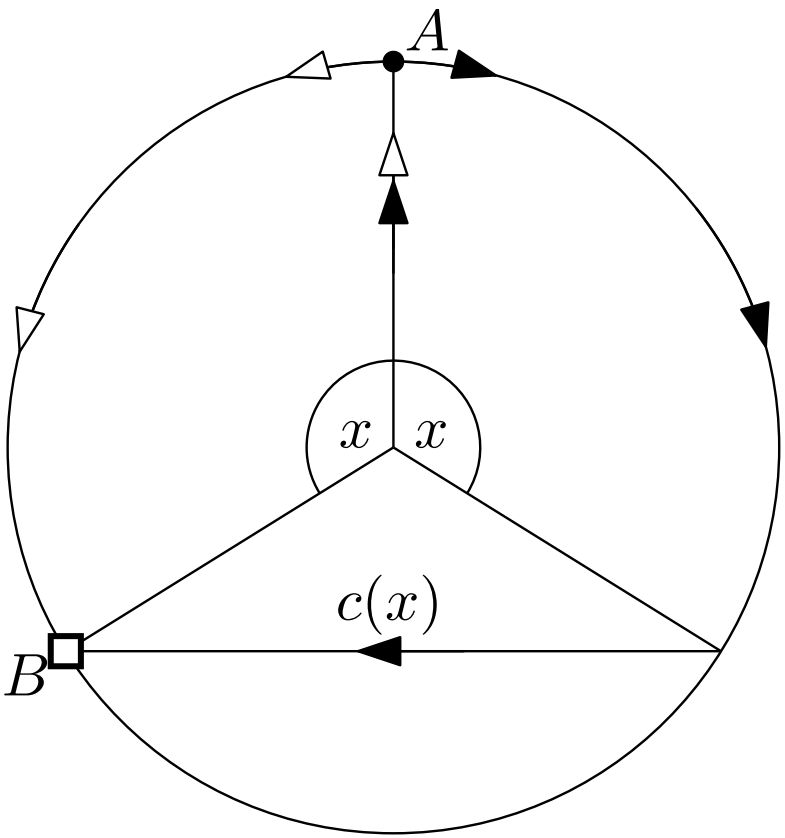
- Robots go in opposite directions along the perimeter.
- When one robot finds exit, it notifies the other using wireless model.



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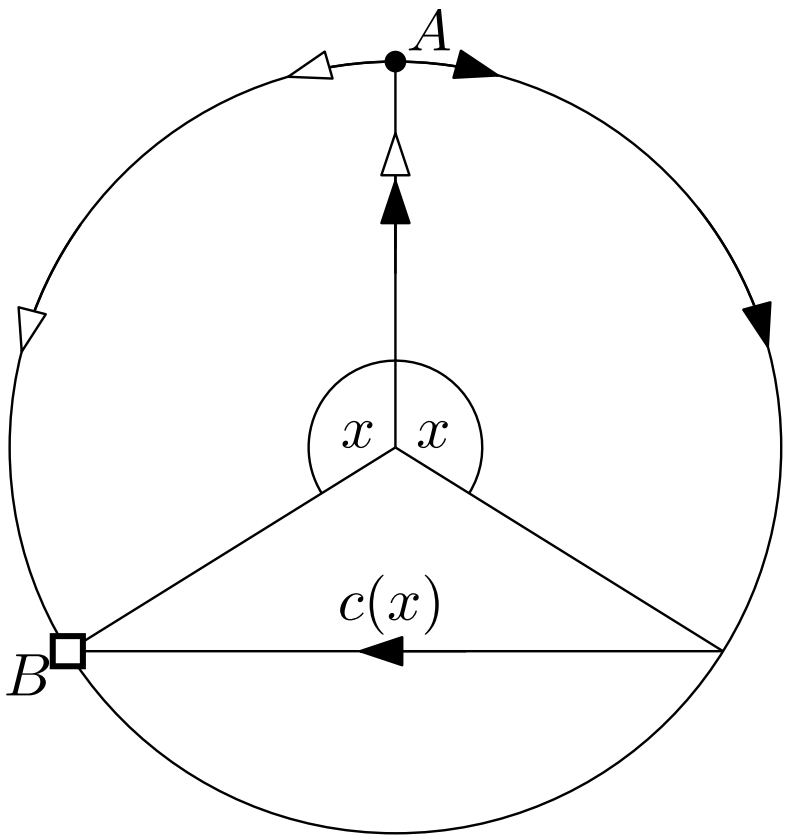
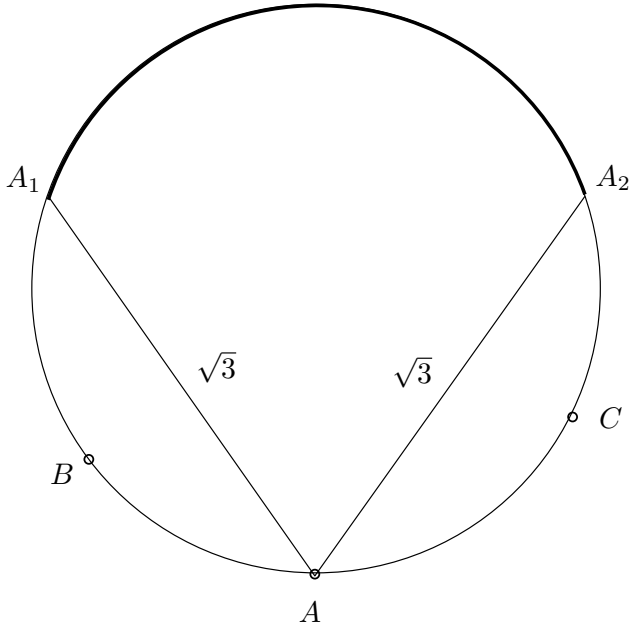



- Evacuation time $T(x) = 1 + x + c(x)$, where $c(x) = 2\sin(\frac{2\pi - 2x}{2}) = 2\sin x$.
- Maximum evacuation time $\max_x T(x) = 1 + 2\pi/3 + \sqrt{3}$.
- This is optimal because any algorithm after time $1 + x$ will have a chord of length $c(x)$ with both endpoints unexplored (Lets see why!).

# Wireless, $k = 2$, lower bound

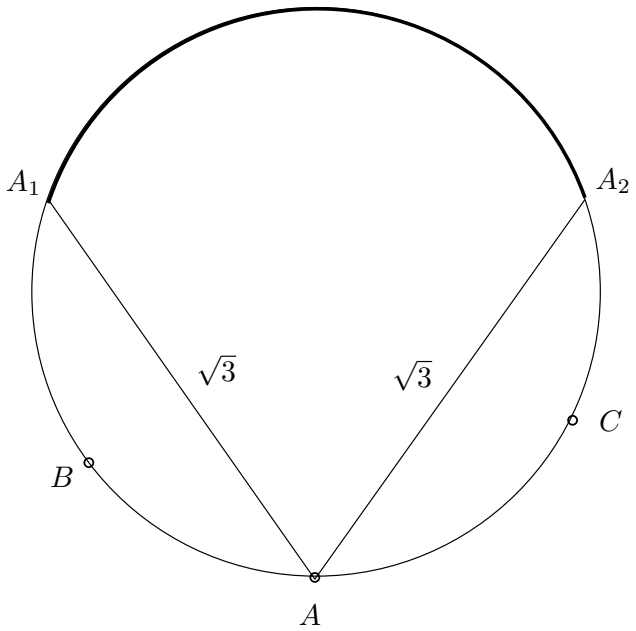
- It takes 1 time unit for the two robots to reach the perimeter.
- In another $2\pi/3$ time the two robots can explore together at most $4\pi/3$ total length of the perimeter.
- This leaves length $\geq 2\pi - 4\pi/3 = 2\pi/3$ of the perimeter unexplored.
- **Claim:** There is a chord of length at least $\sqrt{3}$ none of whose endpoints has been explored by a robot.




- Assume no such chord exists.
- It follows that for any unexplored point $A$ if we draw two chords $AA_1$ and $AA_2$ each of length $\sqrt{3}$ then each point in the arc $\widehat{A_1A_2}$ must be explored by a robot.

# Wireless, $k = 2$, lower bound

- Same observation holds for any unexplored point in either of $\widehat{AA_1}, \widehat{AA_2}$.




- If we consider the leftmost and righmost unexplored points on the circle on either side of $A$, say $B$ and $C$, then their distance must be at least $\sqrt{3}$ (if $|BC| < \sqrt{3}$ then the explored portion would exceed $4\pi/3$, a contradiction).

- Take this pair of points and consider a chord connecting them.
- Such a chord has length $\geq 2\sin(\pi/3) = \sqrt{3}$ and has both endpoints unexplored: adversary can place the exit in any of two endpoints.
- Consider the moment when some robot visits the first endpoint of the chord: the adversary places the exit in the other endpoint and such robot will have to walk at least the length of the chord.
- The total evacuation time is at least $1 + 2\pi/3 + \sqrt{3}$.

# Large $k$ summary

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| Non-wireless | large $k$ | $3 + \frac{2\pi}{k}$ | $3 + \frac{2\pi}{k} - O(k^{-2})$ |
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# Non-wireless, large $k$, algorithm

## Algorithm

- Robots divide the perimeter among themselves (into arcs of size $\frac{2\pi}{k}$).
- Each robot goes to its arc, explores it, and returns to the center.
- The robot that found the exit tells the others about the location of the exit.

## Theorem

It is possible to evacuate $k$ robots in the non-wireless model in time $3 + \frac{2\pi}{k}$.

# Non-wireless, large $k$, lower bound

## Lower bound

For any $k$ and $1 < \alpha < 2$, evacuation takes time at least $\min\left\{3 + \frac{\alpha\pi}{k}, 3 + 2\sin\left(\pi - \frac{\alpha\pi}{2}\right)\right\}$ in the non-wireless model.

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## Proof

Take three time intervals $I_1 = [0, 1), I_2 = [1, 1 + \frac{\alpha\pi}{k}), I_3 = [1 + \frac{\alpha\pi}{k}, 3)$. Consider trajectories of "not disturbed" robots.

- Case 1: Trajectory of robot $r$ is in point $p$ on the perimeter during interval $I_3$. Place the exit in point $p + \pi$, antipodal to $p$. It will take time at least $3 + \frac{\alpha\pi}{k}$ for $r$ to evacuate.

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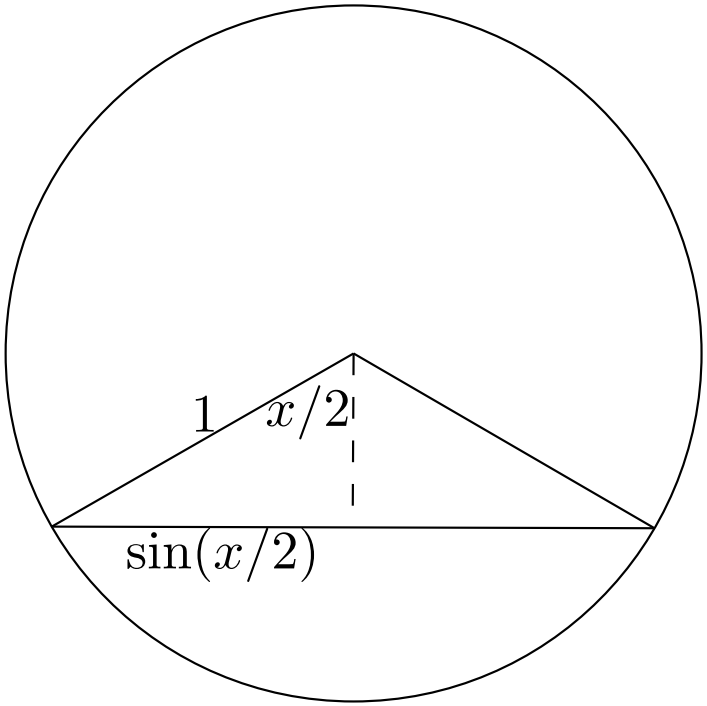
- Case 2: None of the trajectories is on the perimeter in interval $I_3$. Robots were exploring only in interval $I_2$. At time 3, $2\pi - \alpha\pi$ space on the perimeter is not visited. There exists a chord of length $2\sin(\pi - \frac{\alpha\pi}{2})$ with both endpoints not visited (Lemma about chord).

# Non-wireless, large $k$, lower bound




**Lemma about chord**

If a total length of $x + \epsilon$ of perimeter is not explored then there exists a chord on length $2\sin(x/2)$ with both endpoints unexplored (for any $\epsilon > 0$).

**Theorem**

For any $k$, evacuation in the non-wireless model in the worst case takes time at least $3 + \frac{2\pi}{k} - O(k^{-2})$.

**Proof.**

Set $\alpha = 2k/(k+1)$ in the "Lower bound". □

# Wireless, large $k$, lower bound

**Theorem**

For any $k$, evacuation in the wireless model in the worst case takes time at least $3 + \frac{\pi}{k}$.

**Proof**

- Within time $1 + \frac{\pi}{k}$ less than half of the perimeter is explored.
- There exists a diameter with both endpoints unexplored.
- When a robot visits one endpoint we put the exit in the other.
- Total time of $3 + \frac{\pi}{k}$ is needed.

# Wireless, large $k$, algorithm

## Algorithm

- Team $\alpha$ of size $k^{2/3}$ and $\beta$ of size $k - k^{2/3}$.
- $\alpha$ is assigned part of the perimeter of size $\pi - 2\sqrt{\pi}/k^{1/3}$.
- $\beta$ is assigned the rest.
- Teams divide their parts among themselves, explore them.
- When team $\beta$ is done, robots start moving towards the center.
- When any robot discovers the exit, it notifies others via wireless and all robots go to the exit.

## Theorem

The algorithm works in time at most $3 + \frac{\pi}{k} + O(k^{-4/3})$.

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  - Wireless helps.
  - Difference in evacuation time becomes insignificant when $k$ gets big.
- Some open problems:
  - Tighter bounds for small $k$ ($k = 2, 3, 4$).
  - Robots in arbitrary starting positions.

# Conclusions and open problems

- There is separation between wireless and non-wireless communication:
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  - Difference in evacuation time becomes insignificant when $k$ gets big.
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  - Tighter bounds for small $k$ ($k = 2, 3, 4$).
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  - Randomization helps significantly in the cow-path problem – it can reduce the expected search time from $9d$ to $4.59d$. Can it help in the multi-robot evacuation?

# Main Published Work on Evacuation

- J. Czyzowicz, L. Gasieniec, T. Gorry, E. Kranakis, R. Martin, D. Pajak, Evacuating Robots from an Unknown Exit Located on the Perimeter of a Disc. In proceedings of DISC 2014, Oct 12-15, Austin, Texas.
- J. Czyzowicz, K. Georgiou, E. Kranakis, L. Narayanan, J. Opatrny, B. Vogtenhuber, Evacuating Robots from a Disk Using Face-to-Face Communication. In proceddings of CIAC 2015, May 20-22, Paris.

Thank You!